CLASS 9 – MATHEMATICS (NCERT)

AVERAGES OF AVERAGES, MIXTURE AND WEIGHTED MEAN

(Cheat Sheet)



1 AVERAGES OF AVERAGES

Key Concept

- The average of a group of averages is a weighted mean of the individual averages, where the weights are the number of items in each group.
- **Formula:**

$$\text{Overall average} = \frac{n_1a_1 + n_2a_2 + \ldots + n_ka_k}{n_1 + n_2 + \ldots + n_k}$$

where $a_1, a_2, \ldots, a_k$ are the group averages and $n_1, n_2, \ldots, n_k$ are the corresponding numbers of items.

Example

The average marks of 10 students in a class is 70, and the average marks of the next 10 students is 80. What is the average marks of the 20 students?

Solution:

$$\text{Overall average} = \frac{10(70) + 10(80)}{10 + 10}$$

$$= \frac{700 + 800}{20} = 75$$

Important Point

- The number of items in each group acts as the weight.

2 MIXTURE

Key Concept

- When two or more quantities of the same kind are mixed, the average (or resulting value) is the weighted mean, where the weights are the quantities (or amounts).
- **Formula** (for two quantities):

$$\text{Resulting value} = \frac{q_1x_1 + q_2x_2}{q_1 + q_2}$$

where q_1, q_2 are the quantities and x_1, x_2 are the corresponding values.

Example

A vessel contains 40 litres of a 20% sugar solution. Another 60 litres of a 40% sugar solution is added. What is the concentration of the resulting solution?

Solution:

$$\text{Resulting concentration} = \frac{40(20) + 60(40)}{40 + 60}$$

$$= \frac{800 + 2400}{100} = 32\%$$

Important Point

- The quantity (amount) acts as the weight.
- The units of the value must be the same.
- The final value is often expressed in the same unit (%, price, concentration, etc.).

3 WEIGHTED MEAN

Key Concept

- When different items have different importance (weights), the average is called a weighted mean.
- **Formula:**

$$\text{Weighted mean} = \frac{\sum(w_ix_i)}{\sum w_i}$$

where w_i are the weights and x_i are the corresponding values.

Example

A student gets the following marks:
Test 1: 80 (weight 1)
Test 2: 70 (weight 2)
Test 3: 90 (weight 1)
Find the weighted mean.

Solution:

$$\text{Weighted mean} = \frac{1(80) + 2(70) + 1(90)}{1 + 2 + 1}$$

$$= \frac{80 + 140 + 90}{4} = 77.5$$

Important Point

- Weights can be marks, time, quantity, cost, etc.
- If all weights are equal, the weighted mean is the simple average.

4 IMPORTANT POINTS

- Average of averages is a weighted mean.
- In a mixture, the quantity is the weight.
- In a weighted mean, the weights may be different for each value.
- The sum of weights is the total weight.
- The weighted mean lies between the smallest and largest values (if all weights are positive).
- Always keep the units consistent (e.g., %, marks, litres, money).

5 COMMON VARIATIONS

Averages of Averages

1. Find the average of three groups with given averages and numbers.
2. Find the overall average when some group sizes are given.

Mixture

3. Find the resulting percentage after mixing two or more solutions.
4. Find the value after mixing quantities with different prices or concentrations.

Weighted Mean

5. Find the weighted mean when different weights are given.
6. Find the value if one of the weights or values is unknown.

6 QUICK REVISION

- ✓ Use $\frac{n_1a_1 + n_2a_2 + \ldots + n_ka_k}{n_1 + n_2 + \ldots + n_k}$ for averages of averages.
- ✓ Use $\frac{q_1x_1 + q_2x_2}{q_1 + q_2}$ for mixtures.
- ✓ Use $\frac{\sum(w_ix_i)}{\sum w_i}$ for weighted mean.
- ✓ Always match the units.
- ✓ The result lies between the smallest and largest values (when weights/quantities are positive).

7 KEY TAKEAWAY

Different situations, same idea — Weighted Mean!

Think ★ Apply ★ Score Higher