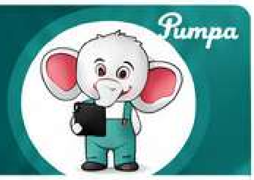
A SQUARE AND A CUBE

Squares, Cubes & Their Beautiful Patterns



1. KEY IDEAS AT A GLANCE

- **Square:** $n \times n = n^2$ (read as "n squared")
- **Cube:** $n \times n \times n = n^3$
- **Perfect squares:** 1, 4, 9, 16, 25, ...
- **Perfect cubes:** 1, 8, 27, 64, 125, ...
- $\sqrt{n}$ = square root (positive value)
- $\sqrt[3]{n}$ = cube root (positive value)

Important!

- A perfect number cannot be a perfect square.
- Use properties to identify non-squares quickly.
- Cubes follow their own patterns.

2. SQUARE NUMBERS (n^2)

n (1–10)	n^2	n (11–20)	n^2	n (21–30)	n^2
1	$1^2 = 1$	11	$11^2 = 121$	21	$21^2 = 441$
2	$2^2 = 4$	12	$12^2 = 144$	22	$22^2 = 484$
3	$3^2 = 9$	13	$13^2 = 169$	23	$23^2 = 529$
4	$4^2 = 16$	14	$14^2 = 196$	24	$24^2 = 576$
5	$5^2 = 25$	15	$15^2 = 225$	25	$25^2 = 625$
6	$6^2 = 36$	16	$16^2 = 256$	26	$26^2 = 676$
7	$7^2 = 49$	17	$17^2 = 289$	27	$27^2 = 729$
8	$8^2 = 64$	18	$18^2 = 324$	28	$28^2 = 784$
9	$9^2 = 81$	19	$19^2 = 361$	29	$29^2 = 841$
10	$10^2 = 100$	20	$20^2 = 400$	30	$30^2 = 900$

3. CUBE NUMBERS (n^3)

n (1–10)	n^3	n (11–20)	n^3
1	$1^3 = 1$	11	$11^3 = 1331$
2	$2^3 = 8$	12	$12^3 = 1728$
3	$3^3 = 27$	13	$13^3 = 2197$
4	$4^3 = 64$	14	$14^3 = 2744$
5	$5^3 = 125$	15	$15^3 = 3375$
6	$6^3 = 216$	16	$16^3 = 4096$
7	$7^3 = 343$	17	$17^3 = 4913$
8	$8^3 = 512$	18	$18^3 = 5832$
9	$9^3 = 729$	19	$19^3 = 6859$
10	$10^3 = 1000$	20	$20^3 = 8000$

4. PROPERTIES & PATTERNS OF PERFECT SQUARES

Unit Digit Pattern

Unit digit of n	Unit digit of n^2
1 or 9	1
2 or 8	4
3 or 7	9
4 or 6	6
5	5
0	0

- Numbers ending in 2, 3, 7 or 8 cannot be perfect squares.
- 00 Perfect squares can have only an even number of zeros.
- Even number → even square. Odd number → odd square.

Differences of Consecutive Squares

$(n+1)^2 - n^2 = 2n + 1$ (odd)

$2^2 - 1^2 = 3$

$3^2 - 2^2 = 5$

$4^2 - 3^2 = 7$

$5^2 - 4^2 = 9$

...

Sum of First n Odd Numbers

$1 + 3 + 5 + ... + (2n-1) = n^2$

$1 = 1^2$

$1 + 3 = 4 = 2^2$

$1 + 3 + 5 = 9 = 3^2$

$1 + 3 + 5 + 7 = 16 = 4^2$

...

Between Consecutive Squares

Between n^2 and $(n+1)^2$, there are 2n non-square numbers.

Example: between 2^2 and 3^2 (4 and 9) → $2 \times 2 = 4$ numbers (5, 6, 7, 8)

Squares & Triangular Numbers

Sum of any two consecutive triangular numbers is a perfect square.

1 + 3 → 3 + 6 → 6 + 10 → 10 + 15

$4 = 2^2$ | 9 = 3^2 | [illegible] = 4^2 | $5^2 = 9^2$

Successive Differences of Squares

1 4 9 16 25 36 ...

↓ ↓ ↓ ↓ ↓ ↓

3 5 7 9 11 ...

↓ ↓ ↓ ↓ ↓

2 2 2 2 2 ...

5. PROPERTIES & PATTERNS OF PERFECT CUBES

Unit Digit Pattern

Unit digit of n	Unit digit of n^3
0	0
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9

Other Cube Properties

- Cube of positive number → positive.
- Cube of negative number → negative.
- Even^3 → even cube.
- Odd^3 → odd cube.
- $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$ $(b \neq 0)$
- $(\text{Decimal})^3$ → decimal places are tripled.
- Cube cannot end with exactly two zeros (zeros are in multiples of 3).

Cubes & Consecutive Odd Numbers

n^3 = sum of n consecutive odd numbers starting from $n(n-1)+1$

$1 = 1 = 1^3$

$3 + 5 = 8 = 2^3$

$7 + 9 + 11 = 27 = 3^3$

$13 + 15 + 17 + 19 = 64 = 4^3$

$21 + 23 + 25 + 27 + 29 = 125 = 5^3$

$31 + 33 + 35 + 37 + 39 + 41 = 216 = 6^3$

...

Successive Differences of Cubes

1 8 27 64 125 216 ...

↓ ↓ ↓ ↓ ↓ ↓

7 19 37 61 91 ... Level 1 (odd numbers)

↓ ↓ ↓ ↓ ↓

12 18 24 30 ... Level 2

↓ ↓ ↓ ↓

6 6 6 6 ... Level 3 (all same)

Prime Factorisation Method (Cube Root)

1. Write number as product of prime factors.
2. Group factors in triplets.
3. If no factor is left over → perfect cube. Otherwise → not a perfect cube.
4. Take one factor from each triplet and multiply to get cube root.

Example: $3375 = 3^3 \times 5^3 = (3 \times 3 \times 3)(5 \times 5 \times 5)$

Cube root $= 3 \times 5 = 15$

6. CHECK: PERFECT SQUARE OR NOT?

Prime Factorisation Method

1. Write number as product of prime factors.
2. Group factors in pairs.
3. If no factor is left over → perfect square. Otherwise → not a perfect square.
4. Multiply one from each pair → $\sqrt{n}$.

Example: $36 = 2 \times 2 \times 3 \times 3 = (2 \times 2)(3 \times 3)$

$= 2^2 \times 3^2 \rightarrow \sqrt{36} = 2 \times 3 = 6$

7. CHECK: PERFECT CUBE OR NOT?

Prime Factorisation Method

1. Write number as product of prime factors.
2. Group factors in triplets.
3. If no factor is left over → perfect cube. Otherwise → not a perfect cube.
4. Multiply one from each triplet → $\sqrt[3]{n}$.

Example: $3375 = 3^3 \times 5^3 = (3 \times 3 \times 3)(5 \times 5 \times 5)$

$\sqrt[3]{3375} = 3 \times 5 = 15$

8. SQUARE ROOT & CUBE ROOT

Square Root ($\sqrt{n}$)

- Inverse of square.
- $\sqrt{n} = x$ means $x^2 = n$
- Every perfect square has two integer roots: $\pm x$
- In this chapter, we consider positive root only.
- $\sqrt{n} > 0$

Methods

1. Prime factorisation
2. Repeated subtraction of odd numbers
3. Using closest squares (estimation)

Cube Root ($\sqrt[3]{n}$)

- Inverse of cube.
- $\sqrt[3]{n} = x$ means $x^3 = n$
- Cube root is unique.
- $\sqrt[3]{n}$ can be negative, zero or positive.

Methods

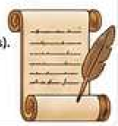
1. Prime factorisation
2. Estimation using nearest cubes

9. HISTORY HIGHLIGHT

Ancient Babylonians listed squares & cubes (~1700 BCE) for practical use.

In India, terms *varga* (square) and *ghana* (cube) were used from early times.

The word "root" comes from Sanskrit *mula* (root, origin, basis).



QUICK FORMULA BOX

Squares

$n^2 = n \times n$

$(n + 1)^2 = n^2 + 2n + 1$

$(n - 1)^2 = n^2 - 2n + 1$

Sum of first n odd numbers $= n^2$

$(n + 1)^2 - n^2 = 2n + 1$

$\sqrt{n} = x \Rightarrow x^2 = n \ (x \geq 0)$

Cubes

$n^3 = n \times n \times n$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$n^3 - m^3 = (n - m)(n^2 + nm + m^2)$

$\sqrt[3]{n} = x \Rightarrow x^3 = n$

MUST REMEMBER

- ✔ Squares end in 0, 1, 4, 5, 6, 9.
- ✔ Squares can only have an even number of zeros.
- ✔ Difference of consecutive squares is an odd number.
- ✔ Sum of first n odd numbers $= n^2$.
- ✔ A number is a perfect square if its prime factors can be paired.
- ✔ Cubes end in 0, 1, 8, 7, 4, 5, 6, 3, 2, 9.
- ✔ Cube factors can be grouped in triplets.
- ✔ Cube of even number → even, cube of odd → odd.
- ✔ $\sqrt{n}$ is positive; $\sqrt[3]{n}$ is unique.

IMPORTANT QUESTIONS FREQUENTLY ASKED

1. Find the square root of 1764. — Hint: Use prime factorisation.
2. Which of the following are perfect squares? 1521, 2025, 2500, 4096. — Hint: Check using prime factorisation or end digits.
3. Find the smallest number by which 1728 must be multiplied to make it a perfect cube. — Hint: Prime factors should be in triplets.
4. Find $\sqrt[3]{17576}$. — Hint: Prime factorisation method.
5. Find the smallest square number divisible by 15. — Hint: Use LCM of prime factors with even exponents.
6. Between which two consecutive square numbers does 1500 lie? — Hint: Compare with nearby squares (38^2 and 39^2).
7. If $n^3 - 1$ is divisible by $n - 1$, what is the value of n? — Hint: Use $n^3 - 1 = (n - 1)(n^2 + n + 1)$.
8. How many perfect squares are there between 1 and 1000? — Hint: Count using squares: 1^2 to 31^2.

KEY TAKEAWAY: Squares and cubes are built on simple ideas—multiply a number by itself 2 times or 3 times. Learn the patterns, use prime factorisation to test, and remember: square roots have ± values (but we use +), while cube roots are unique.