

REVISITING IRRATIONAL NUMBERS

Real Numbers



1 CHAPTER SNAPSHOT

- We recall the difference between rational and irrational numbers.
- We prove that $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, etc. are irrational using proof by contradiction.
- We understand how irrational numbers behave when added, subtracted, multiplied or divided with rational numbers.
- We use these ideas to solve problems in real-life contexts.



Key idea:

A number is irrational if it cannot be written in the form $\frac{p}{q}$, where p, q are integers and $q \neq 0$.

2 HIGH-YIELD CONCEPTS

- 1 Rational number:**
A number that can be written in the form $\frac{p}{q}$, where p, q are integers and $q \neq 0$.
- 2 Irrational number:**
A number that cannot be written in the form $\frac{p}{q}$, where p, q are integers and $q \neq 0$.
- 3 Square root of a non-perfect square is irrational:**
For any natural number n which is not a perfect square, $\sqrt{n}$ is irrational.
- 4 Proof by contradiction:**
Assume the number is rational in the form $\frac{p}{q}$ (in lowest terms) and show this leads to a contradiction.
- 5 Operations with irrational numbers:**
Addition, subtraction, multiplication and division of two irrational numbers may or may not be irrational.

3 IMPORTANT RESULTS & FACTS

- **Rational number (standard form):**
 $x = \frac{p}{q}$ where p, q are integers, $q \neq 0$.
- **Irrational number:**
A number that cannot be written in the form $\frac{p}{q}$, where p, q are integers, $q \neq 0$.
- **Square root property:**
If n is not a perfect square, then $\sqrt{n}$ is irrational.
- **Operations (correct statements):**
 1. Addition, subtraction, multiplication and division of two irrational numbers may or may not be irrational.
 2. Addition of a rational and an irrational number is always irrational.
 3. Subtraction of a rational and an irrational number is always irrational.
 4. Multiplication of a rational and an irrational number is always irrational (if the rational number is non-zero).
 5. Division of a rational and an irrational number is always irrational (if the rational number is non-zero).

4 BOARD EXAM FOCUS (HOTSPOTS)

- Prove that $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ are irrational.
- Use proof by contradiction.
- Prove statements involving irrational numbers.
- Determine whether a given expression is rational or irrational.
- Simplify expressions using properties of irrational numbers.
- Solve short answer and case-based questions.

7 COMMON MISTAKES

- ✗ Assuming every non-integer is irrational.
- ✗ Not writing the number in the form $\frac{p}{q}$.
- ✗ Not using proof by contradiction.
- ✗ Assuming $\sqrt{n}$ is irrational for all n (including perfect squares).
- ✗ Confusing operations with irrational numbers (e.g., assuming sum/product of two irrationals is always irrational).
- ✗ Not checking if the rational number is zero in multiplication/division.
- ✗ Calculation errors in algebraic steps.

5 EXAMPLE HOTSPOTS (TYPICAL Qs)

- 1 Prove that $\sqrt{2}$ is irrational.**
Proof: Assume $\sqrt{2} = \frac{p}{q}$, where p, q are integers and $q \neq 0$, and p and q are coprime.
Then $2 = \frac{p^2}{q^2} \Rightarrow p^2 = 2q^2$.
So p^2 is even $\Rightarrow p$ is even ($p = 2k$).
Then $4k^2 = 2q^2 \Rightarrow q^2 = 2k^2 \Rightarrow q$ is even.
Thus p and q are both even, contradicting that they are coprime. Hence, $\sqrt{2}$ is irrational.
- 2 Show that $3 + \sqrt{2}$ is irrational.**
Proof (by contradiction): Assume $3 + \sqrt{2}$ is rational.
Let $3 + \sqrt{2} = \frac{p}{q}$, where p, q are integers and $q \neq 0$.
Then $\sqrt{2} = \frac{p}{q} - 3 = \frac{p - 3q}{q}$, which is rational (difference of two rational numbers).
This contradicts the fact that $\sqrt{2}$ is irrational. Hence, $3 + \sqrt{2}$ is irrational.
- 3 Show that $\frac{5}{\sqrt{2}}$ is irrational.**
Proof (by contradiction): Assume $\frac{5}{\sqrt{2}}$ is rational.
Let $\frac{5}{\sqrt{2}} = \frac{p}{q}$, where p, q are integers and $q \neq 0$.
Then $\sqrt{2} = \frac{5q}{p}$, which is rational (quotient of two rational numbers). This contradicts the fact that $\sqrt{2}$ is irrational. Hence, $\frac{5}{\sqrt{2}}$ is irrational.



Note:

The above results are useful when working with expressions involving irrational numbers.

6 SMART REVISION

- ✓ Know the definitions of rational and irrational numbers.
- ✓ Remember that $\sqrt{n}$ is irrational when n is not a perfect square.
- ✓ Use proof by contradiction.
- ✓ Addition of a rational and an irrational number is always irrational.
- ✓ Subtraction of a rational and an irrational number is always irrational.
- ✓ Multiplication of a rational and an irrational number is always irrational (if the rational number is non-zero).
- ✓ Division of a rational and an irrational number is always irrational (if the rational number is non-zero).
- ✓ Practice NCERT examples and exercises from this topic.



KEY TAKEAWAY

- ▶ A number is irrational if it cannot be written in the form $\frac{p}{q}$, where p, q are integers and $q \neq 0$.
- ▶ Addition, subtraction, multiplication and division of two irrational numbers may or may not be irrational.
- ▶ Addition (or subtraction) of a rational and an irrational number is always irrational.

