



1. KEY CONCEPTS

- A polynomial in x is an expression of the form $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, where $a_n, a_{n-1}, \dots, a_0$ are real numbers and $a_n \neq 0$.
- The degree of a polynomial is the highest power of x .
- Polynomials of degrees 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.
- A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most 3 zeroes.

★ In general, any polynomial of degree n can have at most n zeroes.

2. TYPES OF POLYNOMIALS

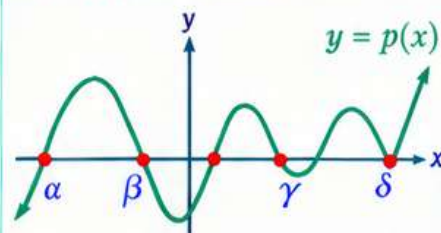
Degree	Gege	General form ($a_n \neq 0$)	Example
1	Linear polynomial	$ax + b$	$2x - 3$
2	Quadratic polynomial	$ax^2 + bx + c$	$x^2 - 5x + 6$
3	Cubic polynomial	$ax^3 + bx^2 + cx + d$	$x^3 + 3x - 1$
4	Polynomial of degree n	$a_n x^n + \dots + a_0$	$x^4 - 2x^2 + 1$ (degree 4)

IMPORTANT

A polynomial of degree n can have at most n zeroes. It may have less than n zeroes.

3. GEOMETRIC MEANING OF ZEROES

The zeroes of a polynomial $p(x)$ are precisely the x -coordinates of the points where the graph of $y = p(x)$ intersects the x -axis.



In this example, the graph intersects the x -axis at four points, so the zeroes are α, β, γ and δ . (Polynomial of degree 4 can have at most 4 zeroes.)

4. ZERO OF A LINEAR POLYNOMIAL

If α is the zero of $p(x) = ax + b$, then

$$\alpha = \frac{-b}{a} = -\frac{(\text{constant term})}{(\text{coefficient of } x)}$$

Example:

For $p(x) = 3x - 9$,

$$\text{zero } \alpha = \frac{-(-9)}{3} = 3$$

5. ZEROES OF A QUADRATIC POLYNOMIAL

If α, β are the zeroes of $p(x) = ax^2 + bx + c$, then

$$\alpha + \beta = \frac{-b}{a} = -\frac{(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\alpha\beta = \frac{c}{a} = \frac{(\text{constant term})}{(\text{coefficient of } x^2)}$$

Example:

For $p(x) = x^2 - 5x + 6$,

$$\alpha + \beta = 5, \quad \alpha\beta = 6 \quad (\text{zeroes: } 2, 3)$$

6. ZEROES OF A CUBIC POLYNOMIAL

If α, β, γ are the zeroes of

$p(x) = ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = \frac{-b}{a} = -\frac{(\text{coefficient of } x^2)}{(\text{coefficient of } x^3)}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{(\text{coefficient of } x)}{(\text{coefficient of } x^3)}$$

$$\alpha\beta\gamma = \frac{-d}{a} = -\frac{(\text{constant term})}{(\text{coefficient of } x^3)}$$

Example:

For $p(x) = x^3 - 6x^2 + 11x - 6$,

$$\alpha + \beta + \gamma = 6, \quad \alpha\beta + \beta\gamma + \gamma\alpha = 11, \quad \alpha\beta\gamma = 6 \quad (\text{zeroes: } 1, 2, 3)$$

7. GENERAL FORMS USING ZEROES

Quadratic polynomial (zeroes α, β)

$$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

More generally: $p(x) = k[x^2 - (\alpha + \beta)x + \alpha\beta]$ ($k \neq 0$)

Cubic polynomial (zeroes α, β, γ)

$$p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$$

More generally:

$$p(x) = k[x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma]$$
 ($k \neq 0$)

8. QUICK EXAMPLES

1 Find the zero of $p(x) = 5x + 10$.

$$\text{Solution: } \alpha = \frac{-10}{5} = -2$$

2 If α, β are zeroes of $p(x) = x^2 + 4x - 5$, find $\alpha + \beta$ and $\alpha\beta$.

$$\text{Solution: } \alpha + \beta = -4, \quad \alpha\beta = -5.$$

9. COMMON MISTAKES

- ✗ Using the wrong formula for sum or product of zeroes.
- ✗ Confusing the coefficients and constant term.
- ✗ Assuming a quadratic has more than 2 zeroes or a cubic has more than 3 zeroes.
- ✗ Ignoring the condition $\alpha \neq 0$.
- ✗ Taking a graph with more than x x -intercepts for a polynomial of degree n .
- ✗ Writing the general form without the correct signs.
- ✗ Thinking zeroes are the y -coordinates instead of x -coordinates.

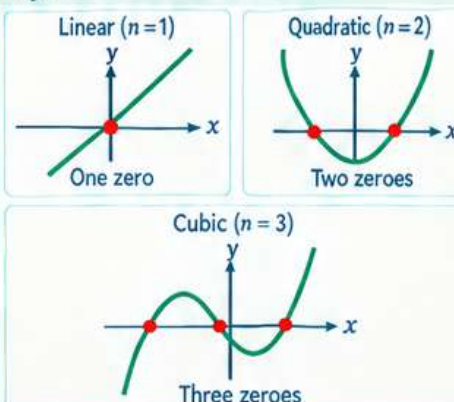
10. KEY TAKEAWAYS

- Zeroes are the x -coordinates where the graph cuts the x -axis.
- Use the relations between zeroes and coefficients to find unknowns quickly.
- A polynomial of degree n can have at most n zeroes.
- The number of zeroes equals the number of x -intercepts (at most n).
- A polynomial may have less than n zeroes.
- Always check your answers by substituting the zeroes in $p(x)$.



PRACTICE • THINK • SCORE!

11. VISUAL SUMMARY



12. RAPID REVISION (30 SECONDS)

- Degree: $n \Rightarrow$ at most n zeroes.
- Linear: $\alpha = \frac{-b}{a}$.
- Quadratic: $\alpha + \beta = \frac{-b}{a}, \quad \alpha\beta = \frac{c}{a}$.
- Cubic: $\alpha + \beta + \gamma = \frac{-b}{a},$
 $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a},$
 $\alpha\beta\gamma = \frac{-d}{a}.$
- General forms using zeroes.
- Zeroes = x -intercepts of the graph.
- Check: substitute zeroes in $p(x)$.
- Maximum number of x -intercepts = degree.